B.Sc - Part I

Notes - 3

Th. 7
$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)$$

$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx. \quad -(1)$$

In the second integral on the right hand side, put $x = 2a-y$, then $dx = -dy$, when $x = a$, $y = a$ then $x = 2a$, $y = 0$

$$\therefore \int_a^{2a} f(x)\,dx = -\int_a^0 f(2a-y)\,dy$$

$$= \int_0^a f(2a-y)\,dy = \int_0^a f(2a-x)\,dx$$

Substituting this in (1) we get the required result.

Th. 8 If $f(x)$ is a periodic function

i.e. $f(x) = f(a \pm x)$

then $$\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$$

$$\int_0^{na} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx + \int_{2a}^{3a} f(x)\,dx + \text{———} \text{———} + \int_{(n-1)a}^{na} f(x)\,dx. \quad \text{—(1)}$$

In the 2nd integrand on the right of (1) if we put $x = a+y$ we get

$$\int_a^{2a} f(x)\,dx = \int_0^a f(a+y)\,dy$$

$$= \int_0^a f(a+x)\,dx = \int_0^a f(x)\,dx$$

Similarly $$\int_{2a}^{3a} f(x)\,dx = \int_a^a f(a+y)\,dy$$

$$= \int_a^{2a} f(a+x)\,dx$$

$$= \int_a^{2a} f(x)\,dx = \int_0^a f(x)\,dx$$

and so on

Hence (1) gives

$$\int_0^{na} f(x)\,dx = n\int_0^a f(x)\,dx$$

Hence $\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx$

if $f(2a-x) = f(x)$ and $= 0$ if $f(2a-x) = -f(x)$