

Th. 7

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$$

$$\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx \quad \text{--- (1)}$$

In the second integral on the right hand side, put $x = 2a - y$, then $dx = -dy$, when $x = a$, $y = a$ then $x = 2a$, $y = 0$

$$\begin{aligned} \therefore \int_a^{2a} f(x) dx &= - \int_a^0 f(2a-y) dy \\ &= \int_0^a f(2a-y) dy = \int_0^a f(2a-x) dx \end{aligned}$$

Substituting this in (1) we get the required result.

Th. 8 If $f(x)$ is a periodic function

i.e. $f(x) = f(a+x)$
then

$$\int_0^{na} f(x) dx = n \int_0^a f(x) dx$$

$$\int_0^{na} f(x) dx = \int_0^a f(x) dx + \int_a^{2a} f(x) dx + \int_{2a}^{3a} f(x) dx + \dots + \int_{(n-1)a}^{na} f(x) dx \quad \text{--- (1)}$$

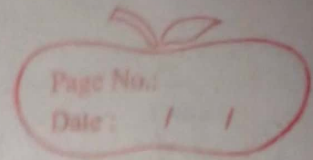
In the 2nd integrand on the right of (1) if we put $x = a+y$ we get

$$\int_a^{2a} f(x) dx = \int_0^a f(a+y) dy$$

$$= \int_0^a f(a+x) dx = \int_0^a f(x) dx$$

Similarly $\int_{2a}^{3a} f(x) dx = \int_a^{2a} f(a+y) dy$

$$= \int_a^{2a} f(2a-x) dx$$



$$= \int_a^{2a} f(x) dx = \int_0^a f(x) dx$$

and so on

Hence (1) gives

$$\int_0^{na} f(x) dx = n \int_0^a f(x) dx$$

$$\text{Hence } \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx$$

$$\text{if } f(2a-x) = f(x) \text{ and } = 0$$

$$\text{if } f(2a-x) = -f(x)$$